

Research Seminars in

Mathematical Finance

Stochastic Volatility Models,
Option Pricing,
Calibration



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Preface

This ebook contains a set of slides that can be used to present lectures in a graduate course or in a research seminar in mathematical finance.

The ebook is divided in three chapters, each chapter is divided in three sections, each section contains a set of slides that can be used to present a lecture. The lectures are independent one from the others.

The lectures discuss the results of the research in mathematical finance of the authors during the years 2007-2012. The results presented are concerned with the study of stochastic volatility models (Heston, SABR models and their generalizations), and in particular with the option pricing and calibration problems relative to these models.

These results have been published in papers appeared in academic journals. The papers are complemented with multimedia material and mathematical software available on the web.

To each lecture are associated:

1. a research paper,
2. a website,
3. a set of slides.

The papers and the websites are listed at the beginning of the sections and are already of public domain. This ebook makes of public domain the slides.

A general reference to the work of the authors and of their co-authors in mathematical finance is the website: <http://www.econ.univpm.it/recchioni/finance>.

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