4

Antenna/Relay for Coded Cooperative Networks

In this chapter, we analyze the impact of antenna/relay selection on the performance of cooperative networks in conjunction with the distributed coding scheme introduced in Chapter 3. Of course, antenna/relay selection can be used with any other coding scheme and any relay configuration. For simplicity, we assume that there is a single relay that is equipped with n_R antennas and only the best antenna is selected. The selection criterion is based on selecting the best source-relay sub-channel (out of the n_R sub-channels at the relay). The latter assumption is needed essentially to preserve the original structure of distributed MIMO systems where each relay node is assumed to be equipped with one antenna and one RF chain. For this scenario, assuming DF and AF relaying, we derive upper bounds on the BER for M−PSK transmission. Our analytical results show that the proposed scheme achieves full diversity for the entire range of bit error rate of interest, unlike the case without antenna selection.

As for relay selection, in terms of performance analysis, it is exactly the same as the case for antenna selection provided that the receive antennas see independent fades. A main difference between the two cases is that, with relay selection, the relays need to feedback their reliabilities to the source to decide on what relay to use, whereas this feedback is not required for antenna selection. On the other hand, with relay selection, the problem of spatial correlation that arises from collocated antennas can be avoided when the relays are equipped with single antennas. One can also consider multiple antennas and multiple relay selection.

4.1 System Model and Preliminaries

The system model considered in this section is the same one considered in Chapter 3 except that the relay node here is assumed to be equipped with n_R antennas. As shown in Figure 4.1, for simplicity, there are three nodes: source, relay and destination. The transmitter is equipped with two recursive systematic convolutional (RSC) encoders, denoted by E_1 and E_2, whose rates are $R_{C_1} = K/N_1$ and $R_{C_2} = K/N_2$, respectively, where K is the information sequence length, N_1 is the codeword length at the output of E_1, and N_2 is the codeword length at the output of E_2. The information sequence b is encoded by E_1, resulting in X_1. Assuming M−PSK, X_1 is then modulated, resulting in a modulated sequence, C_1, denoted as Frame 1. The length of this sequence is $n_1 = N_1/\log_2 M$. C_1 is then broadcasted from the source to the relay and destination nodes. At the relay, the signal corresponding to the best source-relay sub-channel (out of the n_R received signals) is selected.

4.1.1 Decode-and-Forward Relaying

The system model considered, depicted in Figure 4.1, is the same one considered in Chapter 3 except that the relay node here is assumed to be equipped with n_R antennas.

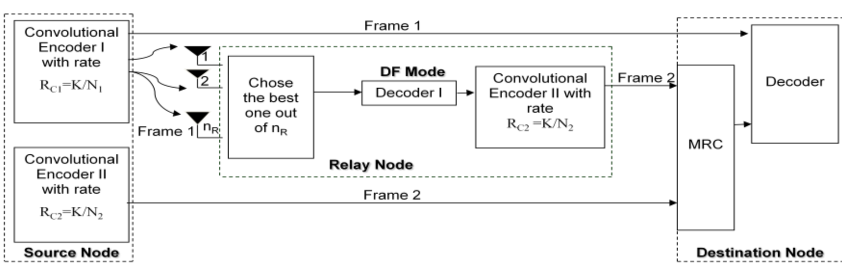

Figure 4.1 *Distributed coded transmission scheme in the DF mode with receive antenna selection at the relay node.*

Therefore, the signals received at the relay and the destination nodes at time t, in the first frame, are respectively given by

$$y_{SR_j}(t) = \sqrt{R_{C_1}E_{SR}}h_{SR_j}(t)s(t) + w_{SR_j}(t), t = 1,2,\cdots,n_1;\ j = 1,2,\cdots,n_R, \quad (4.1)$$

$$y_{SD}(t) = \sqrt{R_{C_1} E_{SD}} h_{SD}(t) s(t) + w_{SD}(t), \; t = 1,2,\cdots,n_1 \tag{4.2}$$

where *s(t)* is the output of the modulator of the source node at time *t*, $h_{SR_j}(t)$ is the fading coefficient between the source transmit antenna and the *j*th receive antenna at the relay, $h_{SD}(t)$ is the fading coefficient for the $S \rightarrow D$ link, E_{SR} and E_{SD} represent the transmitted signal energies for the corresponding link, $w_{SR_j}(t)$ and $w_{SD}(t)$ are AWGN samples with zero mean and variance $N_0/2$ per dimension, and R_{C_1} is the code rate of convolutional encoder I.

In the second frame, If the relay correctly decodes the message it received from the source, the destination receives two versions of $C2$, one directly from the source and the other from the relay. MRC combining is performed at the destination. In this case, the received signals at the destination node at time *t* are given by

$$y_{RD}(t) = \sqrt{R_{C_2} \alpha E_{RD}} h_{RD}(t) \hat{s}(t) + w_{RD}(t), t = n_1 + 1, n_1 + 2, \cdots, n_1 + n_2, \tag{4.3}$$

$$y_{SD}(t) = \sqrt{R_{C_2} (1 - \alpha) E_{SD}} h_{SD}(t) s(t) + w_{SD}(t), t = n_1 + 1, n_1 + 2, \cdots, n_1 + n_2 \tag{4.4}$$

where $\hat{s}(t)$ is the output of the modulator of the relay node at time *t*, $h_{RD}(t)$ is the fading coefficient of the *R-D* link, E_{RD} represents the transmitted signal energy for the *R- D* link, $w_{RD}(t)$ is an AWGN noise sample with zero mean and variance $N_0/2$ per dimension, and $0 \leq \alpha \leq 1$ is the fraction of power transmitted from the relay node during the second frame. For example, when $\alpha = 0$, the relay node does not transmit (no cooperation); whereas when $\alpha = 1$, the source node does not transmit. That is the relay node transmits with energy αE_{RD}, and the source node transmits with energy $(1 - \alpha) E_{SD}$.

4.1.2 Amplify-and-Forward Relaying

The system model for the AF mode is depicted in Figure 4.2. Let $h_{SR}(t)$ be a vector representing the fading coefficients between the source and the n_R antennas at the relay node, that is,

$$\mathbf{h}_{SR}(t) = \left[h_{SR}^{1}(t), h_{SR}^{2}(t), \cdots, h_{SR}^{n_R}(t)\right].$$

Also let $h_{SR}^{max}(t)$ denote the fading coefficient in $\mathbf{h}_{SR}(t)$ that has the *largest norm*. As such, the signals received at the destination and the relay (after antenna selection) during the first frame are respectively given by

$$y_{SR}(t) = \sqrt{R_{C_1} E_{SR}} h_{SR}^{max}(t) s(t) + w_{SR}(t), \quad t = 1,2,\cdots,n_1, \tag{4.5}$$

$$y_{SD}(t) = \sqrt{R_{C_1} E_{SD}} h_{SD}(t) s(t) + w_{SD}(t), \quad t = 1,2,\cdots,n_1, \tag{4.6}$$

where s(t) is the output of the modulator of the source node at time t.

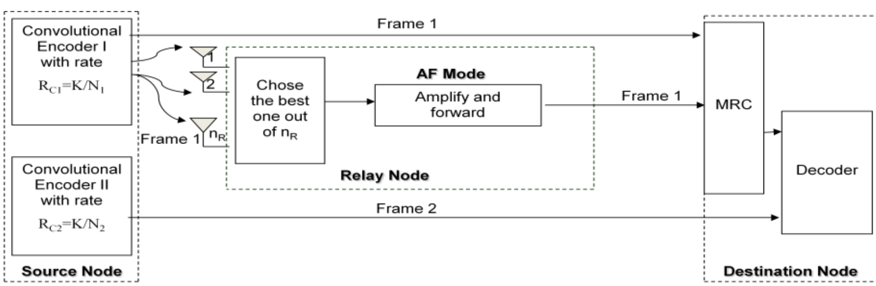

Figure 4.2 *Distributed coded transmission scheme in the AF mode with receive antenna selection at the relay node.*

In the second frame, the destination receives a copy of Frame 1 (i.e., a copy of C_1) from the relay after amplification, as well as Frame 2 directly from the source.

The latter frame is the output of E_2, denoted as X_2, after being modulated to result in a modulated sequence denoted by C_2 of length $n_2=N_2/log_2M$. These two frames are assumed to be transmitted on orthogonal sub-channels. Thus, assuming negligible delay at the relay, the signals received at the destination during the second frame are then given by

$$y_{RD}(t) = A_{RD}(t)h_{RD}(t)y_{SR}(t) + w_{RD}(t), \ \ t = 1,2,\cdots,n_1, \tag{4.7}$$

$$y_{SD}(t) = \sqrt{R_{C_2}(1-\alpha)E_{SD}}h_{SD}(t)s(t) + w_{SD}(t), \ \ t = n_1+1, n_1+2,\cdots,n_1+n_2 \tag{4.8}$$

Where $A_{RD}(t)$ is the amplification factor at the relay. One choice for the gain that we use in this book is [22]

$$A_{RD}(t) = \sqrt{\frac{R_{C_1}\alpha E_{RD}}{R_{C_1}E_{SR}\left|h_{SR}^{max}(t)\right|^2 + \frac{N_0}{2}}}. \tag{4.9}$$

The two received copies of C_1 are combined at the receiver via MRC, and the resulting frame is augmented with the received version of C_2. The augmented codeword is then fed into a Viterbi decoder matched to both encoders at the source to recover the information bits.

4.2 Decode-and-Forward Relaying: Performance Analysis with Selection

In this section, we evaluate the performance of the above scheme with antenna selection in terms of the average BER at the destination. To derive a closed form expression for the upper bound on the pairwise error probability (PEP) with receive antenna selection at the relay node, we first consider the performance with perfect detection at the relay. We understand that this is rather optimistic and can only be justified under special conditions (i.e., high SNR or un-faded channel between the source and relay), but we use it here as a benchmark for the more realistic case, that is, with decoded errors at the relay. If a relay correctly decodes the message it received from the source node using CRC code, re-encode it with a different code and send it to the destination node, which is more realistic to apply.

4.2.1 Decode-and-Forward with Error-Free Relaying

Under the assumption of error free reception at the relay node, the instantaneous received SNR for the channel from *S* to *D* for the first frame is given by

$$\gamma_D(t) = 2R_{C_1}\frac{E_{RD}}{N_0}|h_{SD}(t)|^2 = 2R_{C_1}\gamma_{SD}(t), \;\; t = 1,2,\cdots,n_1, \tag{4.10}$$

and the instantaneous received SNR for the channels from S to D and R to D for the second frame is given by

$$\gamma_D(t) = 2R_{C_2}\left(\frac{(1-\alpha)E_{SD}}{N_0}|h_{SD}(t)|^2 + \frac{\alpha E_{RD}}{N_0}|h_{RD}(t)|^2\right)$$
$$= 2R_{C_2}\big((1-\alpha)\gamma_{SD}(t) + \alpha\gamma_{RD}(t)\big), \; t = n_1+1, n_1+2,\cdots, n_1+n_2, \tag{4.11}$$

where $\gamma_{SD}(t) = \frac{E_{SD}}{N_0}|h_{SD}(t)|^2, \gamma_{RD}(t) = \frac{E_{RD}}{N_0}|h_{RD}(t)|^2$. To maintain the same average power in the second frame, the relay and source nodes split their powers according to the ratio α.

When the fading coefficients h_{SD}, and h_{RD} are constant over the codeword, the conditional pairwise error probability is given by

$$P(d|\gamma_{SD},\gamma_{RD}) = Q\left(\sqrt{2g_{PSK}\big(\big[R_{C_1}d_1 + (1-\alpha)R_{C_2}d_2\big]\gamma_{SD} + \alpha R_{C_2}d_2\gamma_{RD}\big)}\right), \tag{4.12}$$

where d_1 and d_2 are the Hamming distances corresponding to E_1 and E_2, respectively, where $d = d_1 + d_2$. Using (3.10), we can rewrite (4.12) as

$$P(d|\gamma_{SD},\gamma_{RD}) = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} exp\left(\frac{-g_{PSK}\big(R_{C_1}d_1 + (1-\alpha)R_{C_2}d_2\big)\gamma_{SD}}{\sin^2\theta}\right)$$
$$.exp\left(\frac{-g_{PSK}\,\alpha R_{C_2}d_2\gamma_{RD}}{\sin^2\theta}\right)d\theta. \tag{4.13}$$

The average PEP is then given by

$$P(d) = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}\int_0^{\infty}\int_0^{0} exp\left(\frac{-g_{PSK}\left(R_{C_1}d_1+(1-\alpha)R_{C_2}d_2\right)\gamma_{SD}}{\sin^2\theta}\right)$$

$$.exp\left(\frac{-g_{PSK}\,\alpha R_{C_2}d_2\gamma_{RD}}{\sin^2\theta}\right)p_{\gamma_{RD}}(\gamma_{RD})p_{\gamma_{SD}}(\gamma_{SD})d\gamma_{RD}d\gamma_{SD}d\theta \qquad (4.14)$$

Using (3.13), one can show that (4.14) can be expressed as

$$P(d) = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}\left(1+\frac{g_{PSK}\left(R_{C_1}d_1+(1-\alpha)R_{C_2}d_2\right)\bar{\gamma}_{SD}}{\sin^2\theta}\right)^{-1}$$

$$\left(1+\frac{g_{PSK}\,\alpha R_{C_2}d_2\bar{\gamma}_{RD}}{\sin^2\theta}\right)^{-1}d\theta, \qquad (4.15)$$

where $\bar{\gamma}_{SD} = \frac{E_{SD}}{N_0}E[|h_{SD}|^2]$, $\bar{\gamma}_{RD} = \frac{E_{RD}}{N_0}E[|h_{RD}|^2]$ are the average SNRs.

Noting that when the average SNRs $\bar{\gamma}_{SD}$ and $\bar{\gamma}_{RD}$ are relatively high, (4.15) can be approximated as

$$P(d) \approx \left(\sin\frac{\pi}{M}\right)^{-4}\left(\alpha R_{C_2}d_2\bar{\gamma}_{RD}\right)^{-1}\left(\left(R_{C_1}d_1+(1-\alpha)R_{C_2}d_2\right)\bar{\gamma}_{SD}\right)^{-1}$$

$$.\frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}\sin^4\theta d\theta, \qquad (4.16)$$

which suggests that the diversity order achieved is two when the *S*−*R* is error-free. Having obtained the PEP in (4.16), the BER probability can be upper bounded using (3.16).

4.2.2 Decode-and-Forward with Relay Errors

In this section, we consider the realistic scenario in which the relay may fail to correctly decode the message it received from the source, that is, when it's CRC dose not check. As mentioned before, we assume that the relay is equipped with

n_R antennas and only the best antenna is selected. Without loss of generality, we can assume that [44]

$$\gamma_{SR_1}(t) \leq \gamma_{SR_2}(t) \leq \cdots \leq \gamma_{SR_{n_R}}(t), \tag{4.17}$$

where $\gamma_{SR_j}(t) = \frac{E_{SR}}{N_0}\left|h_{SR_j}(t)\right|^2$ and $j = 1,2,\cdots,n_R$. Knowing that the largest number out of n_R nonnegative numbers is always greater than or equal to the average of these n_R numbers, we have [44]

$$\frac{1}{n_R}\sum_{j=1}^{n_R}\gamma_{SR_j}(t) \leq \gamma_{SR_{n_R}}(t). \tag{4.18}$$

Using (4.17) and (4.18), when the fading coefficients h_{SR_j}, h_{SD}, and h_{RD} are constant over the codeword, the conditional PEP is given by

$$P\left(d|\gamma_{SD},\gamma_{RD},\gamma_{SR_j}\right) = \left[1 - Q\left(\sqrt{\frac{2g_{PSK}R_{C_1}d_1}{n_R}\sum_{j=1}^{n_R}\gamma_{SR_j}}\right)\right]$$

$$.\,Q\left(\sqrt{2g_{PSK}\left(\left[R_{C_1}d_1 + (1-\alpha)R_{C_2}d_2\right]\gamma_{SD} + \alpha R_{C_2}d_2\gamma_{RD}\right)}\right)$$

$$+Q\left(\sqrt{\frac{2g_{PSK}\,R_{C_1}d_1}{n_R}\sum_{j=1}^{n_R}\gamma_{SR_j}}\right)Q\left(\sqrt{2g_{PSK}\left[R_{C_1}d_1 + R_{C_2}d_2\right]\gamma_{SD}}\right). \tag{4.19}$$

Using (3.10), we can rewrite (4.19) as

$$P\left(d|\gamma_{SD},\gamma_{RD},\gamma_{SR_j}\right) = \frac{1}{\pi^2}\int_0^{\frac{(M-1)\pi}{M}}\int_0^{\frac{(M-1)\pi}{M}} exp\left(\frac{-g_{PSK}R_{C_1}d_1}{n_R\sin^2\theta_1}\sum_{j=1}^{n_R}\gamma_{SR_j}\right)$$

$$.\,exp\left(\frac{-g_{PSK}\left[R_{C_1}d_1 + R_{C_2}d_2\right]\gamma_{SD}}{\sin^2\theta_2}\right)d\theta_1 d\theta_2 + \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}$$

$$.\,exp\left(\frac{-g_{PSK}\alpha R_{C_2}d_2\gamma_{RD}}{\sin^2\theta}\right)exp\left(\frac{-g_{PSK}\left[R_{C_1}d_1 + (1-\alpha)R_{C_2}d_2\right]\gamma_{SD}}{\sin^2\theta}\right)d\theta$$

$$-\frac{1}{\pi^2}\int_0^{\frac{(M-1)\pi}{M}}\int_0^{\frac{(M-1)\pi}{M}} exp\left(\frac{-g_{PSK}R_{C_1}d_1}{n_R\sin^2\theta_1}\sum_{j=1}^{n_R}\gamma_{SR_j}\right)exp\left(\frac{-g_{PSK}\alpha R_{C_2}d_2\gamma_{RD}}{\sin^2\theta_2}\right)$$

$$.exp\left(\frac{-g_{PSK}\left[R_{C_1}d_1+(1-\alpha)R_{C_2}d_2\right]\gamma_{SD}}{\sin^2\theta_2}\right)d\theta_1 d\theta_2. \quad (4.20)$$

Using (3.13), the average PEP can then be expressed as

$$P(d)=\frac{1}{\pi^2}\int_0^{\frac{(M-1)\pi}{M}}\int_0^{\frac{(M-1)\pi}{M}}\prod_{j=1}^{n_R}\left(1+\frac{g_{PSK}R_{C_1}d_1\bar{\gamma}_{SR_j}}{n_R\sin^2\theta_1}\right)^{-1}$$

$$\left(1+\frac{g_{PSK}\left[R_{C_1}d_1+R_{C_2}d_2\right]\bar{\gamma}_{SD}}{\sin^2\theta_2}\right)^{-1}d\theta_1 d\theta_2+\frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}$$

$$\left(1+\frac{g_{PSK}\alpha R_{C_2}d_2\bar{\gamma}_{SD}}{\sin^2\theta}\right)^{-1}\left(1+\frac{g_{PSK}\left[R_{C_1}d_1+(1-\alpha)R_{C_2}d_2\right]\bar{\gamma}_{SD}}{\sin^2\theta}\right)^{-1}d\theta$$

$$-\frac{1}{\pi^2}\int_0^{\frac{(M-1)\pi}{M}}\int_0^{\frac{(M-1)\pi}{M}}\prod_{j=1}^{n_R}\left(1+\frac{g_{PSK}R_{C_1}d_1\bar{\gamma}_{SR_j}}{n_R\sin^2\theta_1}\right)^{-1}$$

$$.\left(1+\frac{g_{PSK}\,\alpha R_{C_2}d_2\bar{\gamma}_{RD}}{\sin^2\theta_2}\right)^{-1}\left(1+\frac{g_{PSK}\,\left[R_{C_1}d_1+(1-\alpha)R_{C_2}d_2\right]\bar{\gamma}_{SD}}{\sin^2\theta_2}\right)^{-1}d\theta_1 d\theta_2, \quad (4.21)$$

where $\bar{\gamma}_{SR_j}=\frac{E_{SR_j}}{N_0}E\left[\left|h_{SR_j}\right|^2\right]$. If we assume $\bar{\gamma}_{SD}$, $\bar{\gamma}_{RD}$, and $\bar{\gamma}_{SR_j}=\bar{\gamma}_{SR}$ to be large, then (4.21) can be approximated as

$$P(d)\approx\left(\sin\frac{\pi}{M}\right)^{-2n_R-2}\left(\frac{R_{C_1}d_1\bar{\gamma}_{SR}}{n_R}\right)^{-n_R}\left(\left[R_{C_1}d_1+R_{C_2}d_2\right]\bar{\gamma}_{SD}\right)^{-1}$$

$$\cdot\frac{1}{\pi^2}\int_0^{\frac{(M-1)\pi}{M}}\int_0^{\frac{(M-1)\pi}{M}}\sin^{2n_R}\theta_1\sin^2\theta_2\,d\theta_1 d\theta_2+\left(\sin\frac{\pi}{M}\right)^{-4}$$

$$.\left(\alpha R_{C_2}d_2\bar{\gamma}_{RD}\right)^{-1}\left(\left[R_{C_1}d_1+(1-\alpha)R_{C_2}d_2\right]\bar{\gamma}_{SD}\right)^{-1}\frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}\sin^4\theta d\theta$$

$$-\left(\sin\frac{\pi}{M}\right)^{-2n_R-4}\left(\frac{R_{C_1}d_1\bar{\gamma}_{SR}}{n_R}\right)^{-n_R}\left(\left[R_{C_1}d_1+(1-\alpha)R_{C_2}d_2\right]\bar{\gamma}_{SD}\right)^{-1}$$

$$.\left(\alpha R_{C_2}d_2\bar{\gamma}_{RD}\right)^{-1}\frac{1}{\pi^2}\int_0^{\frac{(M-1)\pi}{M}}\int_0^{\frac{(M-1)\pi}{M}}\sin^{2n_R}\theta_1\sin^4\theta_2\, d\theta_1 d\theta_2 \qquad (4.22)$$

By substituting (4.22) into (3.16), one can obtain an upper bound on the probability of bit error.

If we assume $\bar{\gamma}_{SD}=\bar{\gamma}_{RD}=\frac{E_b}{N_0}$, and $\bar{\gamma}_{SR}$ to be large, then *P*(*d*) in (4.22) is the same as *P*(*d*) given in (4.16), which proves that the diversity order is two. However, it should be noted that when $\bar{\gamma}_{SR}$ is very small, many decoding errors will be seen at the relay, leading to possible loss in diversity in the SNR range of interest. However, with antenna selection, the value $\bar{\gamma}_{SR}$ of below which the diversity is lost in much smaller than that without antenna selection, as will be demonstrated in the simulation results section.

4.3 Amplify-and-Forward Relaying: Performance Analysis with Selection

As mentioned before, another relaying method is AF, which is a low-complexity alternative to DF relaying. In this section, we derive the BER performance of the underlying system with antenna selection when the relay node operates in the AF mode. The received SNR for one relay node can be obtained by weighting the combination with the respective powers. From Figure 4.2, the instantaneous received SNR for the channels from *S* to *D* and *R* to *D* for the two frames are given by

$$\gamma_Z(t) = |h_{SD}(t)|^2 \frac{R_{C_1}E_{SD}}{\sigma_{SD}^2} + |h_{SR}^{max}(t)A_{RD}(t)h_{RD}(t)|^2 \frac{R_{C_1}E_{SR}}{\sigma_{SRD}^2(t)},$$

$$t = 1,2,\cdots,n_1, \tag{4.23}$$

$$\gamma_Z(t) = |h_{SD}(t)|^2 \frac{R_{C_2}(1-\alpha)E_{SD}}{\sigma_{SD}^2},\ t = n_1+1, n_1+2,\cdots,n_1+n_2 \tag{4.24}$$

where $\sigma_{SRD}^2(t) = \sigma_{RD}^2 + |A_{RD}(t)h_{RD}(t)|^2\sigma_{SR}^2$. In this case, substituting (4.9) in (4.23) leads to

$$\gamma_Z(t) = 2\left(R_{C_1}\gamma_{SD}(t) + \frac{R_{C_1}\gamma_{SR}^{max}(t)R_{C_1}\alpha\gamma_{RD}(t)}{0.5+R_{C_1}\gamma_{SR}^{max}(t)+R_{C_1}\alpha\gamma_{RD}(t)}\right),\ t = 1,2,\cdots,n_1, \tag{4.25}$$

where $\gamma_{SR}^{max}(t) = |h_{SR}^{max}(t)|^2\frac{E_{SR}}{N_0}$. At high SNR, (4.25) reduces to

$$\gamma_Z(t) = 2\left(R_{C_1}\gamma_{SD}(t) + R_{C_1}\frac{\gamma_{SR}^{max}(t)\alpha\gamma_{RD}(t)}{\gamma_{SR}^{max}(t)+\alpha\gamma_{RD}(t)}\right),\ t = 1,2,\cdots,n_1. \tag{4.26}$$

Using (4.24) and (4.26), when the fading coefficients h_{SD}, and h_{RD} are constant over the codeword, the conditional PEP is then given by

$$P(d|\gamma_{SD},\gamma_{RD},\gamma_{SR}^{max})$$

$$= Q\left(\sqrt{2g_{PSK}\left(\left[R_{C_1}d_1 + (1-\alpha)R_{C_2}d_2\right]\gamma_{SD} + R_{C_1}d_1\frac{\gamma_{SR}^{max}\alpha\gamma_{RD}}{\gamma_{SR}^{max}+\alpha\gamma_{RD}}\right)}\right)$$

$$= Q\left(\sqrt{2g_{PSK}\left(\left[R_{C_1}d_1 + (1-\alpha)R_{C_2}d_2\right]\gamma_{SD} + R_{C_1}d_1\gamma_{SRD}\right)}\right)$$

$$= Q(\sqrt{2g_{PSK}\gamma_D}), \tag{4.27}$$

where

$$\gamma_D = \left[R_{C_1}d_1 + (1-\alpha)R_{C_2}d_2\right]\gamma_{SD} + R_{C_1}d_1\gamma_{SRD},\ \text{and} \tag{4.28}$$

$$\gamma_{SRD} = \frac{\gamma_{SR}^{max}\alpha\gamma_{RD}}{\gamma_{SR}^{max}+\alpha\gamma_{RD}}. \tag{4.29}$$

Using the MGF-based approach, the average PEP is given by [59]

$P(d)$

$$= \frac{1}{\pi} \int_{0}^{\frac{(M-1)\pi}{M}} \Psi_{\gamma_{SD}} \left(\frac{-g_{PSK} \left[R_{C_1} d_1 + (1-\alpha) R_{C_2} d_2 \right]}{\sin^2 \varphi} \right) \Psi_{\gamma_{SRD}} \left(\frac{-g_{PSK} R_{C_1} d_1}{\sin^2 \varphi} \right)$$

$$= \frac{1}{\pi} \int_{0}^{\frac{(M-1)\pi}{M}} \left(1 + \frac{g_{PSK} \left[R_{C_1} d_1 + (1-\alpha) R_{C_2} d_2 \right] \bar{\gamma}_{SD}}{\sin^2 \varphi} \right)^{-1}$$

$$. \Psi_{\gamma_{SRD}} \left(\frac{-g_{PSK} R_{C_1} d_1}{\sin^2 \varphi} \right) d\varphi. \tag{4.30}$$

In order to find *P*(*d*) in (4.30), one has to find a closed form expression for the MGF of γ_{SRD}.

We define random variable (RV) *Z* as

$$Z \triangleq Z_1 + Z_2 = \frac{1}{\gamma_{SRD}}, \tag{4.31}$$

where $Z_1 = \frac{1}{\gamma_{SR}^{max}}$, and $Z_1 = \frac{1}{\alpha \gamma_{RD}}$, γ_{SR}^{max} and γ_{RD} are independent exponential RVs. Now, we recall some definitions that will be used later to evaluate (4.30).

Definition 1 (PDF and MGF of $Z_1 = 1/\gamma_{SR}^{max}$): Let $\mathbf{h}_{SR}$ be a $1 \times n_R$ channel matrix whose elements are i.i.d complex Gaussian random with mean zero and unit variance. Then the PDF of $Z_1 = 1/\gamma_{SR}^{max}$ can be evaluated with the help of [62] to yield

$$p_{Z_1}(z_1) = \sum_{i=0}^{n_R-1} \frac{n_R}{\bar{\gamma}_{SR} z_1^2} \binom{n_R - 1}{i} (-1)^{n_R-1-i} exp\left(-\frac{n_R - i}{\bar{\gamma}_{SR} z_1} \right) U(z_1), \tag{4.32}$$

and its MGF is given by

$$\Psi_{Z_1}(-s) = \int_{0}^{\infty} p_{Z_1}(z_1) \exp(-s z_1)\, dz_1$$

$$= \frac{2n_R\sqrt{s}}{\sqrt{\bar{\gamma}_{SR}}} \sum_{i=0}^{n_R-1} \binom{n_R-1}{i} \frac{(-1)^{n_R-1-i}}{\sqrt{n_R-i}} K_1\left(\sqrt{\frac{4s(n_R-i)}{\bar{\gamma}_{SR}}}\right), \tag{4.33}$$

where $K_1(\cdot)$ is the first order modified Bessel function of the second kind.

Definition 2 (PDF and MGF of $Z_2 = 1/\alpha\gamma_{RD}$): Given an exponential RV γ_{RD}, the PDF of $Z_2 = 1/\alpha\gamma_{RD}$ can be shown as

$$p_{Z_2}(z_2) = \frac{1}{\alpha\bar{\gamma}_{RD}\, z_2^2} exp\left(-\frac{1}{\alpha\bar{\gamma}_{RD}\, z_2}\right) U(z_2) \tag{4.34}$$

and its MGF is given by

$$\Psi_{Z_2}(-s) = \int_0^{\infty} p_{Z_2}(z_2) \exp(-sz_2)\, dz_2$$

$$= \sqrt{\frac{4s}{\alpha\bar{\gamma}_{RD}}} K_1\left(\sqrt{\frac{4s}{\alpha\bar{\gamma}_{RD}}}\right). \tag{4.35}$$

Definition 3 (MGF, CDF, and PDF of Z): The MGF of $Z = Z_1 + Z_2$, $\Psi_Z(-s)$, is given by

$$\Psi_Z(-s) = \Psi_{Z_1}(-s)\Psi_{Z_2}(-s)$$

$$= \frac{4n_R s}{\sqrt{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}} K_1\left(\sqrt{\frac{4s}{\alpha\bar{\gamma}_{RD}}}\right) \sum_{i=0}^{n_R-1} \binom{n_R-1}{i} \frac{(-1)^{n_R-1-i}}{\sqrt{n_R-i}} K_1\left(\sqrt{\frac{4s(n_R-i)}{\bar{\gamma}_{SR}}}\right). \tag{4.36}$$

The cumulative distribution function (CDF) of Z, $P_Z(z)$, can be shown with the help of [63] as

$$P_Z(z) = l^{-1}\left[\frac{\Psi_Z(-s)}{s}\right] =$$

$$l^{-1}\left[\left[\frac{4n_R}{\sqrt{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}} K_1\left(\sqrt{\frac{4s}{\alpha\bar{\gamma}_{RD}}}\right) \sum_{i=0}^{n_R-1} \binom{n_R-1}{i} \frac{(-1)^{n_R-1-i}}{\sqrt{n_R-i}} K_1\left(\sqrt{\frac{4s(n_R-i)}{\bar{\gamma}_{SR}}}\right)\right]\right]$$

$$= \frac{2n_R}{z\sqrt{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}} \sum_{i=0}^{n_R-1} \binom{n_R-1}{i} \frac{(-1)^{n_R-1-i}}{\sqrt{n_R-i}} exp\left(-\left[\frac{(n_R-i)}{\bar{\gamma}_{SR}} + \frac{1}{\alpha\bar{\gamma}_{RD}}\right]\frac{1}{z}\right)$$

$$.K_1\left(\frac{1}{z}\sqrt{\frac{4(n_R-i)}{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right), \tag{4.37}$$

where $l^{-1}(\cdot)$ denotes the inverse Laplace transform.

Then, the PDF of Z, $p_Z(z)$, using Appendix B, is given by

$$p_Z(z) = \frac{2n_R}{z\sqrt{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\sum_{i=0}^{n_R-1}\binom{n_R-1}{i}\frac{(-1)^{n_R-1-i}}{\sqrt{n_R-i}}$$

$$.exp\left(-\left[\frac{(n_R-i)}{\bar{\gamma}_{SR}}+\frac{1}{\alpha\bar{\gamma}_{RD}}\right]\frac{1}{z}\right)\left\{\left[\frac{(n_R-i)}{\bar{\gamma}_{SR}}+\frac{1}{\alpha\bar{\gamma}_{RD}}\right]K_1\left(\frac{1}{z}\sqrt{\frac{4(n_R-i)}{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)\right.$$

$$\left.+\sqrt{\frac{4(n_R-i)}{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}K_0\left(\frac{1}{z}\sqrt{\frac{4(n_R-i)}{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)\right\}, \tag{4.38}$$

where $K_0(\cdot)$ is the zeroth-order modified Bessel function of the second kind.

From the definition of the RV Z presented in (4.31), we note that $\gamma_{SRD} = 1/Z$.

Then, the PDF of γ_{SRD}, $p_{\gamma_{SRD}}(\gamma_{SRD})$, is given by

$$p_{\gamma_{SRD}}(\gamma_{SRD}) = p_Z(z)z^2\big|_{z=\frac{1}{\gamma_{SRD}}}$$

$$= \frac{2\gamma_{SRD}n_R}{\sqrt{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\sum_{i=0}^{n_R-1}\binom{n_R-1}{i}\frac{(-1)^{n_R-1-i}}{\sqrt{n_R-i}}exp\left(-\left[\frac{(n_R-i)}{\bar{\gamma}_{SR}}+\frac{1}{\alpha\bar{\gamma}_{RD}}\right]\gamma_{SRD}\right)$$

$$.\left\{\left[\frac{(n_R-i)}{\bar{\gamma}_{SR}}+\frac{1}{\alpha\bar{\gamma}_{RD}}\right]K_1\left(\gamma_{SRD}\sqrt{\frac{4(n_R-i)}{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)\right.$$

$$\left.+\sqrt{\frac{4(n_R-i)}{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}K_0\left(\gamma_{SRD}\sqrt{\frac{4(n_R-i)}{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)\right\}. \tag{4.39}$$

The MGF of γ_{SRD}, $\Psi_{\gamma_{SRD}}(-s)$, using Appendix B, can be shown to be

$$\Psi_{\gamma_{SRD}}(-s) = \int_0^\infty p_{\gamma_{SRD}}(\gamma_{SRD}) exp(-s\gamma_{SRD}) d\gamma_{SRD}$$

$$= \frac{4n_R}{\sqrt{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}} \sum_{i=0}^{n_R-1} \binom{n_R-1}{i} \frac{(-1)^{n_R-1-i}}{\sqrt{n_R-i}}$$

$$\cdot\left\{\sqrt{\frac{(n_R-i)}{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}} f_1(s,i) + \frac{1}{2}\left[\frac{(n_R-i)}{\bar{\gamma}_{SR}} + \frac{1}{\alpha\bar{\gamma}_{RD}}\right] f_2(s,i)\right\}, \tag{4.40}$$

where

$$f_1(s,i) = \frac{4}{3}\left(\frac{(n_R-i)}{\bar{\gamma}_{SR}} + \frac{1}{\alpha\bar{\gamma}_{RD}} + s + \sqrt{\frac{4(n_R-i)}{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)^{-2}$$

$$.F\left(2, \frac{1}{2}; \frac{5}{2}; \frac{\left(\frac{(n_R-i)}{\bar{\gamma}_{SR}} + \frac{1}{\alpha\bar{\gamma}_{RD}} + s - \sqrt{\frac{4(n_R-i)}{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)}{\left(\frac{(n_R-i)}{\bar{\gamma}_{SR}} + \frac{1}{\alpha\bar{\gamma}_{RD}} + s + \sqrt{\frac{4(n_R-i)}{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)}\right), \text{ and} \tag{4.41}$$

$$f_2(s,i) = \frac{32}{3}\sqrt{\frac{(n_R-i)}{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\left(\frac{(n_R-i)}{\bar{\gamma}_{SR}} + \frac{1}{\alpha\bar{\gamma}_{RD}} + s + \sqrt{\frac{4(n_R-i)}{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)^{-3}$$

$$.F\left(3, \frac{3}{2}; \frac{5}{2}; \frac{\left(\frac{(n_R-i)}{\bar{\gamma}_{SR}} + \frac{1}{\alpha\bar{\gamma}_{RD}} + s - \sqrt{\frac{4(n_R-i)}{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)}{\left(\frac{(n_R-i)}{\bar{\gamma}_{SR}} + \frac{1}{\alpha\bar{\gamma}_{RD}} + s + \sqrt{\frac{4(n_R-i)}{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)}\right), \tag{4.42}$$

where $F(\cdot,\cdot;\cdot;\cdot)$ is Gauss hypergeometric series.

Substituting (4.40) in (4.30), and assuming that $\bar{\gamma}_{SD}$, $\bar{\gamma}_{RD}$, $\bar{\gamma}_{SR}$ are relatively large, we can approximate (4.30) as

$P(d) \approx$

$$\left(g_{PSK}\left[R_{C_1}d_1 + (1-\alpha)R_{C_2}d_2\right]\bar{\gamma}_{SD}\right)^{-1} \frac{4I(M)n_R}{\sqrt{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}} \sum_{i=0}^{n_R-1} \binom{n_R-1}{i} \frac{(-1)^{n_R-1-i}}{\sqrt{n_R-i}}$$

$$\left\{\sqrt{\frac{(n_R-i)}{\alpha\bar{\gamma}_{SR}\bar{\gamma}_{RD}}} f_1\left(-g_{PSK}R_{C_1}d_1, i\right) + \frac{1}{2}\left[\frac{(n_R-i)}{\bar{\gamma}_{SR}} + \frac{1}{\alpha\bar{\gamma}_{RD}}\right] f_2\left(-g_{PSK}R_{C_1}d_1, i\right)\right\}, \quad (4.43)$$

where $I(M) = \frac{1}{\pi}\int_0^{(M-1)\pi/M} \sin^2\varphi d\varphi$. Note that $I(M)$ is a constant that depends on the type of modulation M. By substituting (4.43) into (3.16), we can get the upper bound.

In the case $\bar{\gamma}_{SD} = \bar{\gamma}_{RD} = \bar{\gamma}_{SR} = E_b/N_0$, (4.43) simplifies to

$$P(d) \approx \left(g_{PSK}\left[R_{C_1}d_1 + (1-\alpha)R_{C_2}d_2\right]\right)^{-1}\left(\frac{E_b}{N_0}\right)^{-2}\frac{4I(M)n_R}{\sqrt{\alpha}}$$

$$\cdot\sum_{i=0}^{n_R-1}\binom{n_R-1}{i}\frac{(-1)^{n_R-1-i}}{\sqrt{n_R-i}}$$

$$\left\{\sqrt{\frac{(n_R-i)}{\alpha\left(\frac{E_b}{N_0}\right)^2}} f_1\left(-g_{PSK}R_{C_1}d_1, i\right) + \frac{0.5}{\frac{E_b}{N_0}}\left[(n_R-i) + \frac{1}{\alpha}\right] f_2\left(-g_{PSK}R_{C_1}d_1, i\right)\right\}, \quad (4.44)$$

which shows that the diversity order is two. However, it should be noted that when $\bar{\gamma}_{SR}$ is very small, there will be a loss in diversity but the system still offers large coding gains relative to the non-cooperative case.

4.4 Relay Selection

In the above analysis, we focused on antenna selection where we assumed only one relay node is present. A natural extension of antenna selection would be relay selection. That is, in case there are several relay nodes present in the network, one can use the best nodes to relay the information to the destination. This alternative may be preferred over antenna selection since it may not always be possible to mount multiple antennas on a single relay node, especially for hand-held wireless devices.

From the relay selection point of view, another advantage is that one need not account for spatial correlation that may arise with collocated antennas. Furthermore, to accomplish relay selection, there should be some form of feedback from the relay nodes to the source to decide on which relay(s) to select, which is not a problem since such information is feedback anyway for other purposes. Further extension would be to perform joint antenna/relay selection.

In terms of performance analysis, assuming perfect feedback information at the source, the analytical results obtained above apply to relay selection in a straight-forward manner. Specifically, when there are L relays available and the best relay is selected, the same BER upper bounds derived above apply to relay selection with n_R replaced by L. Another scenario where these results also apply is that when there are L relay nodes with a total of $n_R \geq L$ antennas mounted on all of the relays while only the best antenna is used. With a little bit of more work, one may also extend these results to the case of multiple relay selection in conjunction with antenna selection.

4.5 Simulation Results

In our simulations, we assume that all sub-channels are independent and quasi-static fading. Only one relay node is assumed which can operate in the DF mode or AF mode. BPSK and QPSK modulations are used. In all simulations, the transmitted frame size is equal to $n_1=n_2=130$ coded bits. We also assume that the $R-D$ and $S-D$ channels have equal SNRs, i.e., $\bar{\gamma}_{SD} = \bar{\gamma}_{RD} = E_b/N_0$, but the $S-R$ SNR, $\bar{\gamma}_{SR}$, can be different. We consider two different convolutional codes, whose generator polynomials in octal form are generally given by $(c_1, c_2, c_3, c_4)_{\text{octal}}$. In our context, this implies that E_1 employs $(c_1, c_2)_{\text{octal}}$ and E_2 employs $(c_3, c_4)_{\text{octal}}$. Specifically we use $(13, 15, 15, 17)_{\text{octal}}$ and $(5, 7, 5, 7)_{\text{octal}}$ [44].

For the former code, RSC E_1 employs $(13, 15)_{\text{octal}}$ and E_2 employs $(15, 17)_{\text{octal}}$. The same holds for $(5, 7, 5, 7)_{\text{octal}}$.

Figure 4.3 shows a comparison between the simulated and analytical BER results using (4.16), (3.16), and (4.22) for the two cases of $\bar{\gamma}_{SR} = 3$ and 7 dB. Code $(13, 15, 15, 17)_{\text{octal}}$ is used with $R_{C_1} = R_{C_2} = 0.5$. To maintain the same average power in the second frame, the source and relay nodes divide their power according to the ratio $\alpha = 0.5$. We also include results for the $n_R = 1$ case (i.e., no antenna selection) (Chapter 3). In addition, we include in the figure, for comparison, results for the non- cooperative case (no relaying) as well as for the error-free DF relaying case. These two cases achieve a diversity of one and two, respectively. As shown in the figure, the diversity degrades due to errors at the relay. The interesting observation here is that the loss in diversity starts to become clear when the $S-R$ channel is less reliable than the $S-D$ and $R-D$ channels. As a matter of fact, under the hypothetical scenario when all channels have equal SNRs, the diversity order is maintained for all range of SNR.

In Figure 4.4, we show a comparison of the simulated and analytical BER results based on the expressions given in (4.16), (3.16), and (4.22) for the two cases of $\bar{\gamma}_{SR} = 3$ and 7 dB. Code $(13, 15, 15, 17)_{\text{octal}}$ is used with $R_{C_1} = R_{C_2} = 0.5$ and $\alpha = 0.5$. In this case, we assume $n_R = 1$ where the best antenna is selected. In contrast with the results shown in Figure 4.3, we can clearly see the positive impact of antenna selection. For example, when $\bar{\gamma}_{SR} = 7$ dB, we see that the diversity is maintained until bit error rate 10^{-5} which provides a gain of more than 5 dB over the case without antenna selection. The same is true for the $\bar{\gamma}_{SR} = 3$ dB where the divergence of the curve from the error-free curve occurs a few decibels later. This clearly demonstrates the importance of using antenna selection.

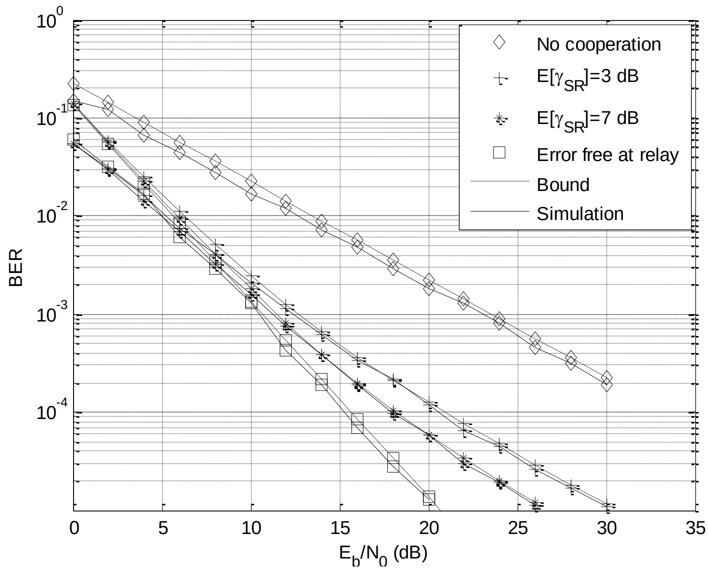

Figure 4.3 *Comparison of analysis and simulated BER for DF relaying over quasi-static fading;* $\bar{\gamma}_{SD} = \bar{\gamma}_{RD} = E_b/N_0$*,* $\bar{\gamma}_{SR} = 3$ *and 7 dB; (13, 15, 15, 17)*$_{octal}$ *with* $R_{C_1} = R_{C_2} = 0.5$*;* $\alpha = 0.5$*;* $n_R = 1$*, i.e., no antenna selection.*

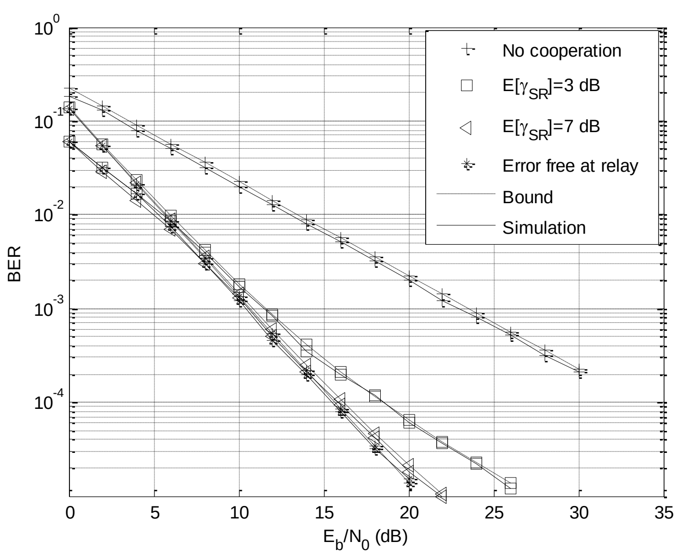

Figure 4.4 *Comparison of analysis and simulated BER for DF relaying over quasi-static fading;* $\bar{\gamma}_{SD} = \bar{\gamma}_{RD} = E_b/N_0$*,* $\bar{\gamma}_{SR} = 3$ *and 7 dB; (13, 15, 15, 17)*$_{octal}$ *with* $R_{C_1} = R_{C_2} = 0.5$*;* $\alpha = 0.5$*;* $n_R = 2$ *and the best antenna is selected.*

Figure 4.5 shows a comparison between the simulated and the bit error rate upper bound corresponding to the expressions given in (3.16), and (4.43) for the two cases of $\bar{\gamma}_{SR} = 3$ and 7 dB. Code $(5, 7, 5, 7)_{\text{octal}}$ is used with $R_{C_1} = R_{C_2} = 0.5$ and $\alpha = 0.5$. We also assume that $n_R = 1$ (no antenna selection). We observe from the figure that the diversity is maintained when all channels ($S-R$, $S-D$, and $R-D$) have equal SNRs, i.e., $\bar{\gamma}_{SR} = \bar{\gamma}_{SD} = \bar{\gamma}_{RD}$. However, the diversity degrades when $\bar{\gamma}_{SR}$ is smaller than the other SNRs, which is similar to the DF relaying case.

To assess the efficacy of antenna selection, we plot in Figure 4.6 the performance of the system corresponding to Figure 4.5 but now with antenna selection. We assume that $n_R = 2$ and the best antenna is used. Similar to the DF case, we observe from the figure that antenna selection preserves the diversity order for a wider range of SNR, which in turns provides substantial coding gains. For instance, for the $\bar{\gamma}_{SR} = 7$ dB and at bit error rate 10^{-5}, there is a gain of about 7 dB when antenna selection is used. Similar favorable results are expected for relay selection, as mentioned before.

Figure 4.7 shows a comparison between the simulated and theoretical BER performance for the two cases of $\bar{\gamma}_{SR} = 4$ and 8 dB, and $M=4$ (QPSK). Code $(5, 7, 5, 7)_{\text{octal}}$ is used with $R_{C_1} = R_{C_2} = 0.5$ and $\alpha = 0.5$. We also assume that $n_R = 2$ and the best antenna is selected. The BER curve when the relay is error-free, having diversity order two is also shown for comparison. It is clear from this figure that as $\bar{\gamma}_{SR}$ gets larger, the performance converges to the ideal error-free case.

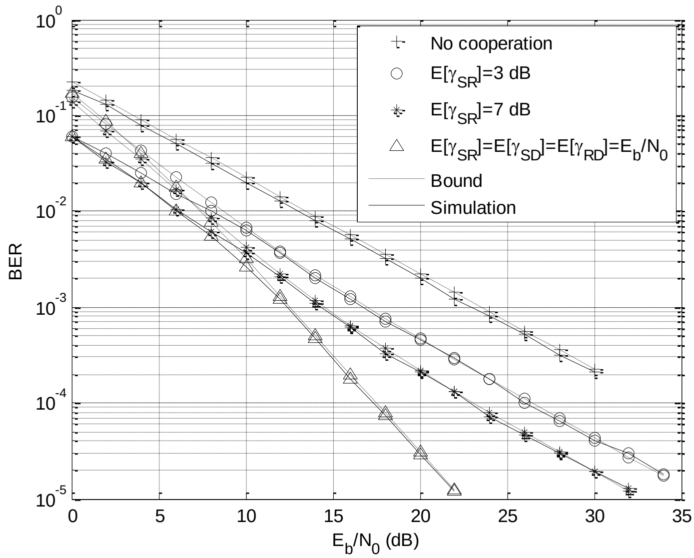

Figure 4.5 *Comparison of analysis and simulated BER for AF relaying over quasi-static fading;* $\bar{\boldsymbol{\gamma}}_{\boldsymbol{SR}} = \mathbf{3}$ *and 7 dB; code (5, 7, 5, 7)*$_{octal}$*;* $\boldsymbol{n_R} = \mathbf{1}$*, i.e., no antenna selection.*

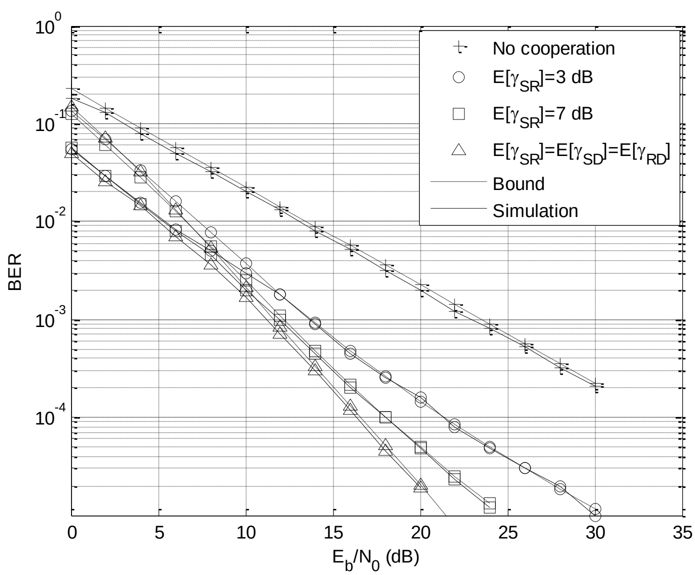

Figure 4.6 *Comparison of analysis and simulated BER for AF relaying over quasi-static fading;* $\bar{\gamma}_{SR} = 3$ *and 7 dB; code (5, 7, 5, 7)*$_{octal}$*;* $n_R = 2$ *and the best antenna is selected.*

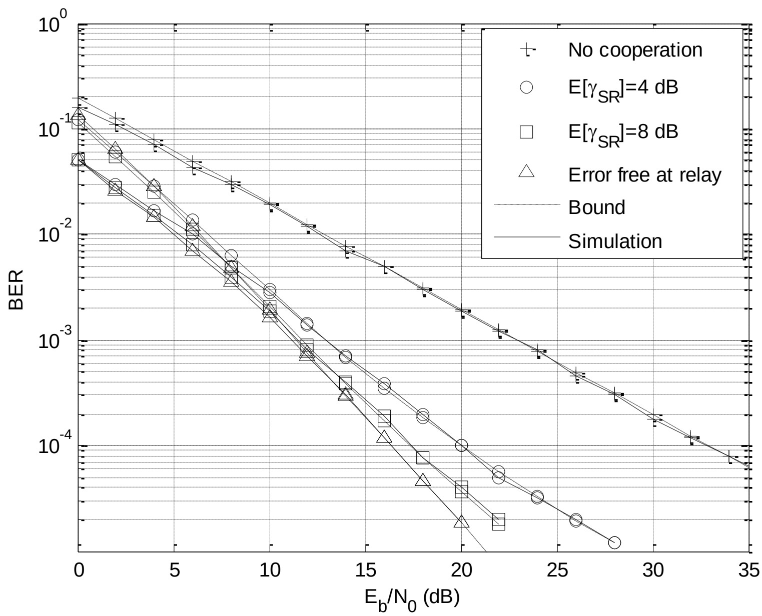

Figure 4.7 *Comparison between the simulated and theoretical BER for QPSK over quasi-static fading, $\bar{\gamma}_{SD} = \bar{\gamma}_{RD} = E_b/N_0$, $\bar{\gamma}_{SR} = 4$ and 8 dB; code $(5, 7, 5, 7)_{octal}$; $n_R = 2$ and the best antenna is selected.*